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**Virginia-STAMP**  
(State Tax Analysis Modeling Program)

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## 2. WHAT IS VIRGINIA-STAMP?

Virginia-STAMP (State Tax Analysis Modeling Program) is a comprehensive model of the Virginia economy, designed to capture the principal effects of state tax changes on that economy. Virginia-STAMP is a five year dynamic computable general equilibrium (CGE) tax model. As such, it provides a mathematical description of the economic relationships among producers, households, government and the rest of the world. It is general in the sense that it takes all the important markets and flows into account. It is an equilibrium model because it assumes that demand equals supply in every market (goods and services, labor and capital); this is achieved by allowing prices to adjust within the model (i.e. they are endogenous). It is computable because it can be used to generate numeric solutions to concrete policy and tax changes, with the help of a computer. And it is a tax model because it pays particular attention to identifying the role played by different taxes.<sup>1</sup>

We begin by distinguishing between producers and consumers. Consumers/households earn income by supplying labor (wages and salaries) and capital (dividends and interest); they also receive transfer payments such as pensions. They are assumed to maximize their utility, which they do by using this income to buy goods and services, pay taxes and save. Their spending decisions are strongly influenced by the structure of prices they face; and the amount of labor that they are willing to provide depends to a substantial degree on the wage rates that they face.

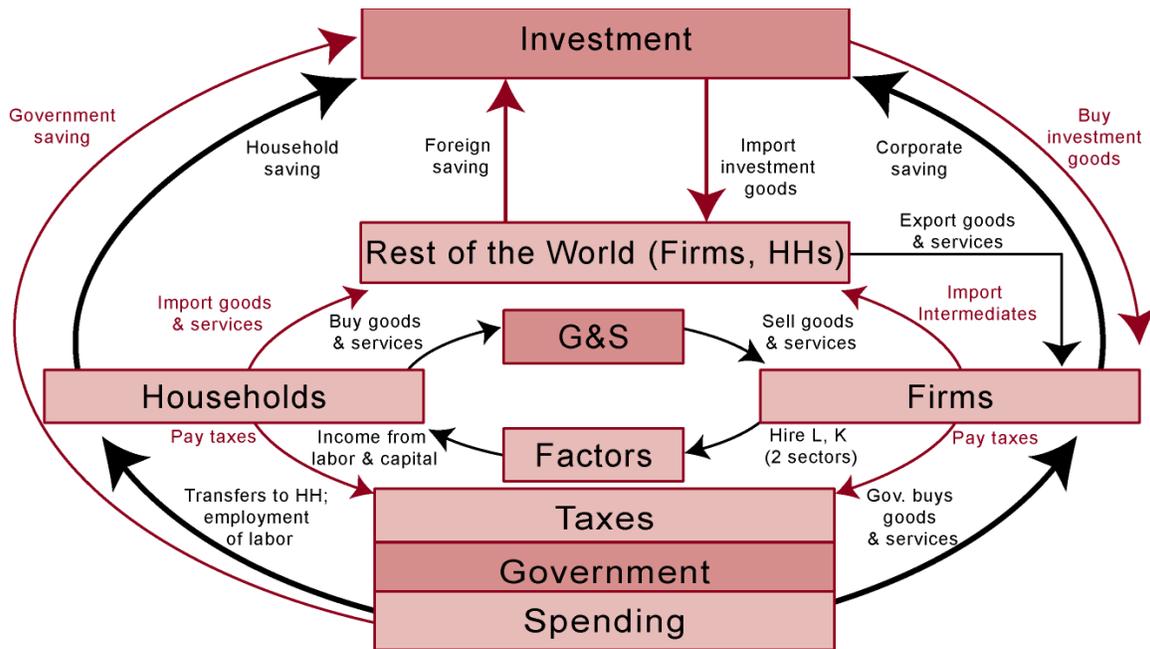
Producers/firms buy inputs (labor, capital and intermediate goods that are produced by other firms) and transform them into outputs. They are assumed to maximize profits and are likely to change their decisions about how much to buy or produce depending on the prices they face for inputs and outputs.

In addition there is a government sector that collects taxes and fees and provides services and transfers. The rest-of-the world sector consists of the entire world outside Virginia. The relationships between these components are set out in the circular flow diagram shown in Figure 1. The arrows in the diagram represent flows of money (for instance, households purchase goods

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<sup>1</sup> For a clear introduction to CGE tax models, see John B. Shoven and John Whalley, "Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey," *Journal of Economic Literature*, XXII (September, 1984), 1008. Shoven and Whalley have also written a useful book on the practice of CGE modeling entitled *Applying General Equilibrium* (Cambridge: Cambridge University Press, 1992).

and services), and flows of goods and services (for instance, households supply their labor to firms). The separate box for government shows the flows of funds to government in the form of taxes, as well as government purchases of goods and services and government hiring of labor and capital.



**Figure 1. Circular Flow Diagram**

Complex as it may seem, the diagram in Figure 1 is still too simple, because it lumps all households into one group, and all firms into another. To provide further detail it is necessary to create *sectors*; Virginia-STAMP has 77 economic sectors. Each sector is an aggregate that groups together segments of the economy. We separate households into seven income classes and firms into 27 industrial sectors. In addition, we distinguish between 22 types of taxes (14 of them at the state level) and 13 categories of government spending. To complete the model there are two factor sectors (labor, capital), an investment sector, four state fund sectors and a sector that represents the rest of the world. The choice of sectors was dictated by the availability of suitably disaggregated data (for households and firms), and the purposes of the model, which is why we provide considerable detail about taxes.

Regional models, such as Virginia-STAMP, are similar in many respects to national and international CGE models. However they differ in a number of important respects, which are worth listing:

- a. In a national model, most saving goes toward domestic investment; however this need not be true at the regional level. If citizens of Virginia save more than they spend, then the excess saving will leak out of the state.
- b. The smaller the unit under consideration, the greater the importance of trade with the rest of the world. This is an important consideration for state models.
- c. Migration is likely to be larger and more responsive across states than across nations.
- d. In regional models, taxes are interdependent. So, for instance, the amount of revenue collected by the Federal personal income tax depends significantly on whether there is a state income tax (which may be deducted from income before computing the Federal tax).
- e. Data are less available at the regional than national level. This explains why scores of national CGE models have been built, but very few regional models.

### ***Constructing a CGE model***

The construction of a CGE model involves several steps. First, one needs to organize the data needed by the model. Virginia-STAMP starts with data for a single year, 2003, which the model uses to develop a steady state path through 2009. This steady state path is attained by applying a growth rate to investment, population, employment and inflation over the time frame of the model. In Virginia-STAMP the investment growth rate is assumed to be 7.44%.<sup>2</sup> The growth rate for population is assumed to be 0.093%.<sup>3</sup> The growth rate for employment is assumed to be 2.2%.<sup>4</sup> The inflation growth rate is assumed to be 1.5%. To attain a reasonable steady state path, the data for the base year, 2003, must be very detailed. Most of the data are organized into a *Social Accounting Matrix (SAM)*, which in this case consists of a 77 by 77 matrix that accounts for the main economic and fiscal flows in the state.

The model also requires some additional information – for instance data on employment and on the structure of the Federal income tax – which are put in separate files. And the model requires information on “elasticities;” these are the parameters, typically gleaned from the academic literature, that measure the responsiveness of households to changes in prices and wages, and of

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<sup>2</sup> This figure is derived from taking the average nominal US gross domestic investment for the period 1929-2002 as published by the Bureau of Economic Analysis.

<sup>3</sup> This figure is the Census projection for Virginia for the period 2005-2010.

<sup>4</sup> This figure represents the average growth rate in employment for Virginia for 1969-2001 as published by the BEA.

firms to changes in input costs and output prices. These are set out in detail in Section 4 of this report. The economy is assumed to be competitive, and to run at full employment (by which we mean that there is no involuntary unemployment).

Second, the model needs to be specified in detail; the next section of this report sets out details of the model that we constructed for Virginia, along with some comments explaining the choices made at each step.

The third step is to program the model. For this we used the specialized GAMS (General Algebraic Modeling System) software. In order to make the model easier to use, we also developed an interface in Microsoft Excel. This allows the user to enter tax changes on an Excel spreadsheet, click on the “Estimate CGE” button, and read the key output on the same spreadsheet; the heavy-duty computing occurs in the background.

Before use, the model has to be calibrated. This consists of running the model – i.e. asking it to solve for all the variables in such a way as to maximize utility, which is the discounted aggregate of state personal income over the time period of the model. The results for the base year are checked to see that they correspond with the actual values of the variables in the SAM (taken to be 2003 in our case). Once the model reproduces the base year values, it is considered calibrated. Calibration is a non-trivial step, and is essentially a way of checking that the model is working properly.

Finally, the model is ready to be used to quantify tax change effects. The procedure is straightforward: specify a new tax rate (or change in the tax), run the model, and compare the new results with the steady state ones. At this point it is also possible to test the sensitivity of the results to different assumptions – such as the values of elasticities – that are incorporated into the model. We note in passing that Virginia-STAMP is a policy model and not a forecasting model; in other words it is designed to answer “what if?” questions, not to estimate what is likely to occur in coming years.

### 3. THE VIRGINIA-STAMP

#### *Organizing the Data*

The starting point in building a CGE model is to determine the degree of detail that is desired and to organize the collected data into the useful format of a SAM for the base year. The SAM that we developed for Virginia is a 77 by 77 matrix. Each of the 5,929 cells represents the dollar value of a flow from one sector of the economy to another – for instance, purchases of business services by the agricultural sector, or labor earnings flowing to middle-income households. Reading along a row one finds the payments received by that sector; reading down a column one sees the payments made by that sector. The SAM is balanced, which means that the sum of the entries in any given row equals the sum of the entries in the corresponding column. Thus, for instance, the revenue received by agriculture must equal spending by that sector, so that all incoming and outgoing funds are completely accounted for.

For Virginia-STAMP, we distinguish 27 industrial sectors, two factors (labor and capital), seven household categories, an investment sector, 39 government sectors (22 for taxes, 13 for spending, 4 government funds) and a sector for the rest of the world. In sectoring the economy we sought to strike a balance between providing a high level of detail (especially on the tax side) and keeping the model to a manageable size. In addition there is a more pragmatic consideration, which is that the lack of finely disaggregated data limits the degree of detail that is possible. Data availability also determined some of the choices we made; for instance, it is possible to get a breakdown of households into seven income categories (see below for further details), and while we might have preferred a different set of categories, we were constrained by the nature of the data available.

#### **Industrial sectors**

Virginia-STAMP contains 27 industrial sectors. Data from the Bureau of Economic Analysis would have allowed us to separate out 49 sectors. However some sectors were too small to merit separate attention, which is why, for instance, we combined some industries, such as textiles and apparel. In some other cases there were no matching employment figures, and so it was easier to work with aggregates. Further, only 37 sectors were distinguished for the input-output table.

### **Factor Sectors**

We distinguish between two factors, labor and capital (which includes land). Businesses pay wages and salaries to labor, and they generate profits. These are then distributed to household owners as factor income.

### **Household Sectors**

In Virginia-STAMP, households receive wages, capital income and transfers; they use this income to buy goods and services; they pay taxes; and they save. We distinguish seven household sectors, which group households by their levels of income. Expenditure data are available for households in each of these categories, which make it relatively straightforward to work with this structure. One purpose of this disaggregation of households is to allow one to trace the distributive effect of tax changes; another is to allow different groups to have different levels of sensitivity to labor market conditions.

### **Investment Sector**

There is one investment/savings sector. Households save, both directly out of their cash incomes, and indirectly because they own shares in businesses that save and reinvest profits. The government also saves and invests. Information is available from the Bureau of Economic Analysis on the pattern of gross investment by destination (i.e. how much gross investment went into adding to the stock of capital in agriculture, in mining, and so on). We have constructed measures of the capital stock in each sector; by applying published depreciation rates and adding gross investment, one arrives at the capital stock in the subsequent period. This permits the model to track the expansion of the economy over time. The BEA has also produced a matrix, built for the U.S. for 1997, that maps investment by destination with investment by source. In other words, it allows one to find out, for instance, how much of the investment destined for agriculture is spent on purchasing goods and services from the construction sector and the transport sector. Thus if investment rises, it is possible to identify which sectors would face an expansion in the demand for their output.

## Government Sectors

Virginia-STAMP was designed primarily to analyze the effects of major changes in the structure of state taxes, and so we have paid particular attention to providing sufficient detail for government transactions. The sectoring is summarized below in Table 3.

<b>Table 1 Government Sectors</b>		
<b>Federal Government Receipts</b>		
USSSTX	Social Security	Receives payments from employers and households; pays out transfers to households.
USPITX	Federal personal income tax	Receives payments from households, which are put into the Federal normal spending account.
USCITX	Federal corporation income tax	Receives payments from corporations and channels them into the Federal normal spending account.
USOTTX	Other federal taxes	Includes excises on motor fuel, alcohol, and tobacco; estate and gift taxes. Also funneled into the Federal normal spending account.
<b>Federal Government Expenditure</b>		
USNOND	Federal normal spending	Federal government purchases goods and services, hires labor, and transfers money to Virginia and to Federal defense fund.
USDEFF	Federal defense spending	Purchases goods and services, and pays labor for military purposes.
<b>Virginia Government Receipts</b>		
STPITX	Virginia individual income tax	Revenues go into Virginia general fund.
STSATX	Virginia Sales Tax	Sales tax, vehicle sales and use tax, utility taxes, hotel and motel tax. Revenues go into Virginia general fund and special.
STFUTX	Virginia tax on motor fuel	Revenues go into Virginia special fund and transportation fund.
STCITX	Virginia corporate income tax	This is the tax on business; revenues go into the Virginia general fund.
STALTX	Virginia tax on alcohol	Revenues go into Virginia general fund.
STMOTX	Virginia tax on motor vehicles	Revenues go into Virginia special fund.
STTCTX	Virginia tax on cigarettes	Revenues go into Virginia general fund.
STIHTX	Virginia tax on insurance occupation	Revenues go into Virginia general fund.
STPSTX	Virginia public service corporation tax	Revenues go into Virginia general fund and other funds.
STMVTX	Tax on motor vehicle purchases	Revenues go into Virginia special fund.
STINTX	Virginia estate tax	Revenues go into Virginia general fund.
STFEES	Virginia fees, licenses, permits	Revenues go into all funds.
STWKTX	Virginia workers' compensation and disability	Sector combines workers compensation and unemployment funds. Receipts go into proprietary fund.
STSPCF	Virginia special fund	An accounting device. Tax revenue is channeled into this fund before being distributed to other uses.
STPROF	Virginia proprietary fund	An accounting device. Tax revenue is channeled into this fund before being distributed to other uses.
STGENF	Virginia general fund	An accounting device. Tax revenue is channeled into this fund before being distributed to other uses.
<b>Virginia Government Expenditure</b>		
STGGSP	Virginia general spending	General government spending.
STEDUC	Virginia spending on education	Mainly purchases of goods and services and labor in the higher education sector.
STHELT	Virginia spending on health & welfare	Buys some services; mainly transfers funds to local health spending fund.
STTRAN	Virginia spending on transport	Mainly buys engineering services and construction.
STPBSF	Virginia spending on public safety	Public safety and fire departments spending.
STOTHS	Virginia other spending	Miscellaneous other spending by the state on labor, goods and services.

<b>Local Government Receipts</b>		
LOPRTX	Local tax on residential property	Collected from households. Transferred to local government spending units.
LOPBTX	Local tax on business property	Collected from firms. Transferred to local government spending units.
LOOTTX	Local taxes, other	Local taxes such as sales tax. Transferred to local government spending units.
<b>Local Government Expenditure</b>		
LOEDUC	Local spending on education	Purchases goods and services and (mainly) pays teacher salaries.
LOHELT	Local spending on health & welfare	Purchases goods and services and pays labor; large transfers to the poorest category of households.
LOTRAN	Local spending on transportation	Mainly buys engineering services and construction.
LOPBSF	Local spending on public safety	Public safety and fire departments local spending.
LOOTHS	Local other spending	Includes spending on police and firefighters, road repair, and miscellaneous local government services.

The Virginia state government collects revenue from taxes on sales, motor fuel, the income tax, excises on alcohol and tobacco, insurance and inheritance. It also collects a variety of fees.

All of the collections from these taxes and fees are deemed to go into one of the following funds, general fund, special fund, proprietary fund or other fund, from whence they flow to different categories of spending.

In the model, the government of Virginia pays directly for some education, mainly the University of Virginia system. It also spends on public safety and transportation and general administration, mostly salaries for state workers. A major category of spending is health and welfare, mostly in the form of transfers to local authorities. All remaining state spending is gathered into a residual category.

Local governments in Virginia receive tax revenue from residential property and business and commercial property, as well as from a variety of other taxes and fees. These funds, augmented by transfers from the state level, flow to spending on education, health and welfare and other spending such as public safety.

### **Rest of the World**

To complete the model we have included a sector for the rest of the world (ROWSCT). This refers to the rest of the United States as well as other countries. Information on flows between Virginia and the rest of the world is difficult to piece together, and is an area where considerable professional judgment was required.

#### **4. VIRGINIA-STAMP IN DETAIL**

In this report we set out the model in detail. First we list our elasticity assumptions used in the model. Second we introduce each equation, providing some context and a short description. Then we present each equation in mathematical form, and finish with information on the sources of data used.

##### ***ELASTICITY ASSUMPTIONS FOR VIRGINIA-STAMP***

The following elasticities are used in industry-specific equations:

ETAM: Import elasticity with respect to domestic price for producers' purchase of intermediates. Most of the data on elasticities are borrowed from Reinert, Roland-Holst, and Shiells. The two most recent are Reinert and Roland-Holst(1992) and Roland-Holst, Reinert and Shiells(1994).

In the first study, the authors estimate an Armington model for 163 mining and manufacturing sectors. Two-thirds of the elasticities were positive and statistically significant, ranging from a low of 0.13 for chocolate to 3.49 for wine, brandy and brandy spirits. The second study looked at the impact of NAFTA. In this study many of the aggregate industries had an elasticity of 1.50. Since import data for goods between states is almost impossible to obtain, we made some assumptions and used 1.50 for most industries and a slightly lower elasticity of 0.50 for a handful of less traded industries such as service industries.

While these elasticities are slightly higher than the literature on national trade, we believe that goods in a state are more price sensitive to goods in the Rest of the World(including other states) than national goods. It is converted to a domestic share elasticity for each industry by virtue of the following equation.  $ETAD = ETAM * IMPORT / (DOM. DEMAND * DOM. SUPPLY SHARE OF DOM. DEMAND)$ . The estimates for this elasticity were taken from the literature.

ETAE: Export elasticity with respect to domestic price for the sale producers' goods. Used in the export demand equation. The NAFTA study was also helpful with exports. We used an elasticity of 1.65 for the industries which had an import elasticity of 1.50 and an export elasticity of 0.65 for those which had an import elasticity of 0.50.

ETAY: Income elasticity of demand for local goods and services. This elasticity appears in the household consumption equation. Estimates were taken from the literature. Literature values range from -0.24 to 2.93.

ETAOP: Cross-price elasticities for goods from different industries. This elasticity appears in the household consumption equation. Estimates were taken from the literature. Literature values from -0.06 to -1.72.

SIGMA: Elasticity of substitution capital and labor. Values in the literature range between 0.15 and 1.809 for industries with the majority close to 1. This measurement is used to calculate RHO, which is the exponent in the production function. The equation is:  $(1 - \text{SIGMA})/\text{SIGMA}$ .

<b>Table 2. Industry Elasticities</b>					
	<b>ETAM</b>	<b>ETAE</b>	<b>ETAY</b>	<b>ETAOP</b>	<b>SIGMA</b>
<b>AGRICF</b>	1.50	-1.65	1.00	-1.00	0.90
<b>MINING</b>	1.50	-1.65	1.00	-1.00	0.80
<b>CONSTR</b>	1.50	-1.65	1.00	-1.00	0.90
<b>FOODPR</b>	1.50	-1.65	1.00	-1.00	0.90
<b>APPARL</b>	1.50	-1.65	1.00	-1.00	0.90
<b>MFRCON</b>	1.50	-1.65	1.00	-1.00	0.80
<b>PPAPER</b>	1.50	-1.65	1.00	-1.00	0.80
<b>CHEMIC</b>	1.50	-1.65	1.00	-1.00	0.80
<b>ELECTR</b>	1.50	-1.65	1.00	-1.00	0.90
<b>MVOTRA</b>	1.50	-1.65	1.00	-1.00	0.90
<b>METALS</b>	1.50	-1.65	1.00	-1.00	0.80
<b>MACHIN</b>	1.50	-1.65	1.00	-1.00	0.80
<b>INSTRU</b>	1.50	-1.65	1.00	-1.00	0.90
<b>MFROTH</b>	1.50	-1.65	1.00	-1.00	0.90
<b>TRANSP</b>	1.50	-1.65	1.00	-1.00	0.90
<b>COMMUN</b>	1.50	-1.65	1.00	-1.00	0.90
<b>UTILIT</b>	1.50	-1.65	1.00	-1.00	0.80
<b>WHOLSA</b>	0.50	-0.65	1.00	-1.00	0.90
<b>RETAIL</b>	0.50	-0.65	1.00	-1.00	0.90
<b>BANKNG</b>	1.50	-1.65	1.00	-1.00	0.90
<b>INSURS</b>	1.50	-1.65	1.00	-1.00	0.90
<b>REALST</b>	1.50	-1.65	1.00	-1.00	0.90
<b>REPSVC</b>	1.50	-1.65	1.00	-1.00	0.80
<b>BSVCS</b>	1.50	-1.65	1.00	-1.00	0.80
<b>ENTRHO</b>	0.50	-0.65	1.00	-1.00	0.80
<b>HEALTH</b>	0.50	-0.65	1.00	-1.00	0.80
<b>OTHSVC</b>	0.50	-0.65	1.00	-1.00	0.80

The following elasticities are used in household-specific equations:

ETAPIT: Labor supply elasticity with respect to taxes. This elasticity appears as an exponent in the labor supply equation. Measurements were based on estimates taken from the literature. The labor supply elasticities are widely divergent in the literature and suffer from a lack of disaggregation. They range from -0.4 to 2.3 for wages, with rather high positive values for women, particularly married woman.

ETATP: Household response to transfer payments. The transfer payment elasticities reflect a study by Robins (1985) on the effects of a negative income tax (NIT). It is also a reflection of observations that income received by upper income groups is unaffected by transfer payments.

ETARA: Labor supply elasticity with respect to average wage. Measurements were derived from literature estimates. This elasticity appears in the labor supply equation.

ETAYD: Responsiveness of immigration to after tax income. Not much literature exists that ties migration to disposable income or unemployment. Studies by Bartik(1991), Valiant(1994), and Treyz et al. (1993) put the range for response to change in wage rates between 0.835 and 2.39. We used these as a basis for our after tax earnings elasticities. This elasticity appears in the population equation.

ETAU: Responsiveness of immigration to unemployment. We made some assumptions based on the responsiveness to employment elasticities in the literature.

ETAMH: Income elasticity of demand for imports by household. This elasticity appears in the household import equation.

	<b>ETAPIT</b>	<b>ETATP</b>	<b>ETARA</b>	<b>ETAYD</b>	<b>ETAU</b>	<b>ETAMH</b>
<b>LESS10</b>	-0.15	-0.05	0.17	1.30	-0.80	0.70
<b>LESS25</b>	-0.18	-0.05	0.17	1.50	-0.80	0.70
<b>LESS50</b>	-0.20	-0.04	0.20	1.60	-0.80	0.70
<b>LESS75</b>	-0.25	-0.04	0.30	1.80	-0.80	0.70
<b>LES100</b>	-0.25	-0.03	0.40	2.00	-0.80	0.70
<b>LES150</b>	-0.30	-0.03	0.50	2.10	-0.80	0.70
<b>MOR150</b>	-0.35	-0.02	0.50	2.30	-0.80	0.70

### **DETAILED EQUATIONS FOR VIRGINIA-STAMP**

Virginia-STAMP is a dynamic CGE model which assumes a steady state growth path. Absent any “shocks”, the economy is assumed to remain on this path. If the economy experiences a shock, such as a tax change, the economy will diverge from this steady state path and eventually return to a new path. The size and length of the divergence will depend on the size of the shock to the economy. Below we set out the equations used in Virginia-STAMP and the assumptions inherent in them.

#### ***HOUSEHOLD DEMAND***

Households are assumed to maximize their well being (“utility”) by picking baskets of goods and services, subject to their budget constraints. The key set of equations in this section is labeled *Private Consumption*, and consists of a set of demand functions. These demand functions, based on a Cobb-Douglas utility function, take on the simple form,

$$X_{t,i} = \lambda_i * \frac{I_t}{P_{t,i}}, \quad i = 1, \dots, n; t = 1, \dots, n,$$

where  $X_{t,i}$  is the quantity demanded of good  $i$  at time  $t$ ,  $P_{t,i}$  is the price of good  $i$  at time  $t$ ,  $I_t$  is income at time  $t$ , and the  $\lambda_i$  are parameters that measure the share of income that is devoted to good  $i$ . This is the simplest specification that is theoretically satisfactory: it is additive (so spending equals income less taxes less saving), has downward-sloping demand (i.e. it ensures that when the price of a good rises the quantity demanded falls), is zero degree homogeneous in prices and income (so that if prices and incomes were to double, the quantity demanded would not change), and meets the technical requirement of symmetry of the Slutsky matrix. More complex formulations are possible, but there is a lack of reliable data on the elasticity parameters that would be needed in such cases.

## Household Gross Factor Income

Comments: The gross income of households in each of the seven groups (indexed by  $h$  in the set  $H$ ) is found by first summing factor income ( $y_f$ ) from labor and capital, subtracting the social security contributions paid by employees, and then allocating the total to each group on the basis of fixed shares. Factor payments are allocated to each household group using the same fixed shares as were found in the base year.

$$\text{Eq.1.} \quad y_{t,h} = \sum_{f \in F} \frac{\alpha_{hf} a_{t,h}^w}{\sum_{h \in H} \alpha_{hf} a_{t,h}^w} y_{t,f} \left( 1 - \sum_{g \in GF} \tau_{t,g,f}^h \right) \quad \forall h \in H$$

Description: Household income is the sum of income from each factor (labor and capital) less factor taxes, distributed by household groups according to their share of total.

Data: The information on earnings for each household group comes from household survey data for the South of the U.S. for 2000-2001. Source: *BLS Consumer Expenditure Report 2001-2002 (South)*.

Available at <ftp://ftp.bls.gov/pub/special.requests/ce/region/y0102/region.txt>

## Household Disposable Incomes

Comments: Disposable household income is gross income, less taxes on household income and property (mainly personal income tax (USPITX, STPITX) and residential property tax (LOPRTX)), plus transfer payments (such as social security and unemployment benefits).

$$\text{Eq.2.} \quad y_{t,h}^d = y_{t,h} - \sum_{g \in GI} t_{t,g,h} a_{t,h}^{gh} - \sum_{g \in GH} \tau_{t,g,h}^h a_{t,h}^{gh} + \sum_{g \in G} w_{hg} a_{t,h}^n \tau_{t,h,g}^{pc} \quad \forall h \in H, t \in T$$

Description: Disposable household income is the household income less income taxes and other household taxes (property taxes etc), plus the government transfer payments.

## Private Consumption Expenditure

Comments: This is the simplest demand system that is consistent with theoretical first principles, and it requires only a limited number of parameters.

$$\text{Eq.3.} \quad c_{t,i,h} = \bar{c}_{t,i,h} \left( \frac{y_{t,h}^d}{\bar{y}_{t,h}^d} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\beta_{ih}} \prod_{i' \in I} \left[ \frac{p_{t,i'}}{\bar{p}_{t,i'}} \left( 1 + \sum_{g \in GS} \tau_{t,g,i'}^c \right) \right]^{\lambda_{ti}} \quad \forall i \in I, h \in H, t \in T$$

Description: Consumption is a function of baseline consumption, adjusted to reflect the change in household disposable income (in constant prices), and the change in after-tax prices.

Data: By construction, this equation has zero cross price elasticities. In the absence of adequate estimates of demand elasticities we follow the approach taken by Berck et al., setting all income and own-price elasticities equal to unity.

## Direct household purchases of imports

Some household spending goes directly to buy goods and services outside Virginia.

$$m_{t,h} = \bar{m}_{t,h} \left( \frac{y_{t,h}^d}{\bar{y}_{t,h}^d} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\eta_h^m} \quad \forall h \in H, t \in T$$

Description: Household imports will increase with the increase in disposable income, in constant prices.

## Household Savings

Comments: In Virginia-STAMP, household savings is the residual after spending and taxes have been subtracted from income. Thus savings are seen as occurring passively.

$$\text{Eq.4.} \quad s_{t,h} = y_{t,h}^d - \sum_{i \in I} c_{t,i,h} p_{t,i} \left( 1 + \sum_{g \in GS} \tau_{t,g,i}^c \right) - m_{t,h} \quad \forall h \in H, t \in T$$

Description: See comments above.

Data: The savings rates for households at each income level were adjusted based on professional judgement, to account for the imputed savings by corporations (which indirectly represents savings by the owners of the corporations).

### Consumer Price Indexes

Comments: The price index in the reference period is set equal to 1. There is a separate price index for each household group. This allows one to compute the real (rather than nominal) income for each household group. For instance, a tax on foodstuffs would tend to hit poor households relatively hard, and the CPI for poor households would pick up this effect.

$$\text{Eq.5.} \quad p_{t,h} = \frac{\sum_{i \in I} p_{t,i} \left( 1 + \sum_{g \in GS} \tau_{t,g,i}^c \right) c_{t,i,h}}{\sum_{i \in I} \bar{p}_{t,i} \left( 1 + \sum_{g \in GS} \tau_{t,g,i}^q \right) c_{t,i,h}} \quad \forall h \in H, t \in T$$

Description: Price index by household group is a function of the baseline price index, adjusted by the change in after-tax prices by industry, according to their corresponding share of consumption.

Data: The consumption of each good by each household group ( $c_{ih}$ ) is derived from Consumer Expenditure Survey data (2000-2001). Expenditures on each product group by household groups were allocated based on the types of products that were reported. For example, expenditures on pork went to the Food sector and expenditures on vehicles went to the Transportation sector (TPORT). The numbers refer to the Northeast region of the US, which we took to be an adequate representation of spending patterns in Virginia. The distribution of households by income group is also for the Northeast rather than Virginia, but we applied the same proportions to the population of Virginia.

## LABOR SUPPLY

Comments: In Virginia-STAMP we model the participation rate, defined as the proportion of households in any given income category that work. The participation rate is assumed to rise if wage rates rise, if the taxes levied on earnings fall, or if the transfer payments paid out per non-working household fall. The participation rate for low-income households is assumed to be highly sensitive to the level of transfer payments, but relatively insensitive to changes in taxes or the wage rate. On the other hand, high-income households are assumed to respond substantially to changes in the taxes and wage rates they face.

$$\text{Eq.6.} \quad a_{t,h}^w = \bar{a}_{t,h}^w \frac{a_{t,h}^{hh}}{\bar{a}_{t,h}^{hh}} \left( \frac{r_{t,L}^a}{\bar{r}_{t,L}^a} \div \frac{P_{t,h}}{\bar{P}_{t,h}} \right)^{\eta_h^{ls}} \left( \frac{\sum_{g \in GI} t_{t,g,h}^{pi}}{\sum_{g \in GI} \bar{t}_{t,g,h}^{pi}} \right)^{\eta_h^{PII}} \left( \frac{\sum_{g \in G} \frac{W_{t,h',g}}{P_{t,h}}}{\sum_{g \in G} \frac{\bar{W}_{t,h',g}}{\bar{P}_{t,h}}} \right)^{\eta_h^{IP}} \quad \forall h \in H, t \in T$$

Description: Supply of labor is a function of baseline supply of labor adjusted by population growth, the net change in wages, income taxes, and government transfer payments.

Data: The data on working households by income class came from the Consumer Expenditure Survey (2000-2001) for the South, as did the total number of households in each category. These were then adjusted to fit the total number of households in Virginia.

## MIGRATION

### Population

Comments: The number of households in each income group depends first and foremost on the initial number of households. To this we add the natural growth of the population and net in-migration. Migration in turn depends on the level of after-tax income, and the proportion of households that are not working (which reflects

the employment prospects facing new migrants). This formulation is in the spirit of the migration model popularized by Harris and Todaro (*American Economic Review*, 1973).

Eq.7. 
$$a_{t,h}^{hh} = \bar{a}_{t,h}^{hh} + \bar{a}_{t,h}^i \left( \frac{y_{t,h}^d}{a_{t,h}^{hh}} \div \frac{\bar{y}_{t,h}^d}{\bar{a}_{t,h}^{hh}} \div \frac{p_{t,h}}{\bar{p}_{t,h}} \right)^{\eta_h^{yd}} \left( \frac{a_{t,h}^n}{a_{t,h}^{hh}} \div \frac{\bar{a}_{t,h}^n}{\bar{a}_{t,h}^{hh}} \right)^{\eta_h^u}$$

$$- \bar{a}_h^o \left( \frac{\bar{y}_{t,h}^d}{\bar{a}_{t,h}^{hh}} \div \frac{y_{t,h}^d}{a_{t,h}^{hh}} \div \frac{\bar{p}_{t,h}}{p_{t,h}} \right)^{\eta_h^{yd}} \left( \frac{\bar{a}_{t,h}^n}{\bar{a}_{t,h}^{hh}} \div \frac{a_{t,h}^n}{a_{t,h}^{hh}} \right)^{\eta_h^u}, \forall h \in H, t \in T$$

Description: See comments above.

Data: The elasticities used in this equation are the same as those used for California by Berck et al. (1996), and “reflect the middle ground found in the literature about migration” (p.117).

### Number of Non-Working Households

Comments: This is a simple accounting equation; the number of non-working households is the total number of households, less the number that are working.

Eq.8. 
$$a_{t,h}^n = a_{t,h}^{hh} - a_{t,h}^w \quad \forall h \in H, t \in T$$

Description: See comments above.

### THE BEHAVIOR OF PRODUCERS/FIRMS

Producers are assumed to maximize profit. Combining intermediate inputs with labor and capital produces output. The amount of intermediate inputs required per unit of output is fixed, but firms have considerable leeway to vary the amounts of capital and labor that they use in production. The value of output less intermediate inputs is value added, and it is useful to compute a price for this value added; it is this price that determines factor demand – i.e. drives firms to hire more or less labor and capital. The amounts of labor and capital inputs, in turn, drive the total value of output via the production function.

## Intermediate Demand

Comments: Intermediate goods constitute a fixed share of the value of production.

$$\text{Eq.9.} \quad v_{t,i} = \sum_{i' \in I} \alpha_{t,i,i'} q_{t,i'} \quad \forall i \in I, t \in T$$

Description: See comments above.

Data: From the Virginia input-output table, derived from data from IMPLAN, which in turn are based on data from by the Bureau of Economic Analysis.

## Production Function

Comments: Output is determined by the quantities of labor and capital used in production; it is assumed that enough intermediate goods will be available. We use a Constant Elasticity of Substitution (CES) production function, which allows a degree of substitution between labor and capital; in other words, if the price of labor rises, firms will cut back on the number of workers they hire, and use more capital instead.

$$\text{Eq.10.} \quad q_{t,i} = \gamma_{t,i} \left[ \sum_{f \in F} \alpha_{t,f,i} (u_{t,f,i}^d)^{-\rho_t} + g \alpha_{t,i} (gk_t)^{-\rho_t} \right]^{-1/\rho_t} \quad \forall i \in I, t \in T$$

Description: In addition to labor and capital used in production, we account for infrastructure.

Data: We use values for the elasticity of substitution that are close to, but slightly lower than, one. This is relatively standard in CGE models. Information on the shares of labor and capital in production come from the Bureau of Economic Analysis.

## Price of Value Added

Comments: Define value-added as the value of output less the cost of intermediate inputs. One may then define a “price” of value added, which we then use below in the factor demand (i.e. labor demand, capital demand) equations.

$$\text{Eq.11.} \quad p_{t,i}^{va} = p_{t,i}^d - \sum_{i' \in I} \alpha_{t,i',i} p_{t,i'} \left( 1 + \sum_{g \in GS} \tau_{t,g,i'}^v \right) \quad \forall i \in I, t \in T$$

Description: Price of value-added by industry is the domestic price by industry minus the production prices by industry according to their share in domestic supply, including taxes on intermediates, if any.

Data: Prices are set equal to unit in the baseline case.

### Factor Demand

Comments: It is possible to construct a profit function that expresses profits as a function of factor inputs. Microeconomic theory shows that the partial first derivative of the profit function, with respect to a given factor demand variable, gives the demand equation for that factor. The left hand side of the equation shows payments to labor (including the cost of factor taxes such as the employer share of social security contributions). The right hand side gives the amount of value added attributable to the factor. There is a separate equation for labor and for capital, for each of the 27 industrial sectors.

$$\text{Eq.12.} \quad r_{t,f,i} r_{t,f}^a \left( 1 + \sum_{g \in GF} \tau_{t,f,g,i}^x \right) u_{t,f,i}^d = p_{t,i}^{va} q_{t,i} \alpha_{t,f,i} \quad \forall i \in I, f \in F, t \in T$$

Description: The factor demand at the current intra-industry rental rate (for labor and capital) times the overall rental rate, including factor taxes is a function of the price of value-added times the industry domestic supply.

Data: Information on the wage bills comes from the Bureau of Economic Analysis. The total wage bill is divided by the numbers of workers (from the Bureau of Labor Statistics) to get measures of wage rates by industry. The intersectoral wage differentials are not allowed to vary within the model. The cost of capital was derived as property-type income divided by the capital stock. The capital stock was constructed by disaggregating the national aggregate level of capital using a series of proxy measures; further details of the methodology are provided

in Appendix 2 of the *Texas State Tax Analysis Modeling Program: Texas-STAMP* (1999) and although this refers to Texas, the same approach was taken in computing the capital stock for Virginia.

### **Factor Income**

Comments: The total income accruing to factors – i.e. to labor and capital – is computed here.

$$\text{Eq.13.} \quad y_{t,f} = \sum_{i \in I} r_{t,f,i} r_{t,f}^a u_{t,f,i}^d + \sum_{g \in G} r_{t,f,g} r_{t,f}^a u_{t,f,g}^d \quad \forall f \in F, t \in T$$

Description: The factor income is the sum of factor demand times rental rates, for all industries and government sectors.

### ***TRADE WITH OTHER STATES AND COUNTRIES***

From a Virginia perspective, the “rest of the world” consists of the remainder of the United States as well as the world outside the U.S. Goods produced in Virginia are assumed to be close, but not perfect, substitutes for goods produced elsewhere. Thus if prices rise in Virginia, the state’s exports will fall and its imports will rise, but the adjustment need not be very large. There is no need for trade to be balanced; capital flows simply adjust to cover the gap between exports and imports. In this section we also develop a measure of the average price faced by domestic households and firms for goods and services produced by each industry, the price is a weighted average of the price of locally produced and imported goods.

### **Demand for Exports**

Comments: Exports depend on the price of goods within the state relative to the price outside Virginia. If the domestic price rises relative to the foreign price, exports will fall. Note that the elasticity here is negative.

Eq.14. 
$$e_{t,i} = \bar{e}_{t,i} \left[ p_{t,i}^d \div \bar{p}_{t,i}^w \right]^{\eta_i^e} \quad \forall i \in I, t \in T$$

Description: Current exports are a function of baseline exports adjusted by the change in domestic prices versus fixed world prices.

Data: The trade data for Virginia are not particularly reliable; we have used our judgement, combined with BEA data, to arrive at sensible estimates. The elasticities we use are similar to those employed by Berck et al.

### **Domestic Share of Domestic Consumption**

Comments: The demand for imports is handled indirectly, by modeling the share of domestic consumption that is supplied by domestic firms ( $d$ ), following the approach pioneered by Armington (1969). This share depends on the domestic price relative to the price of the same goods in the rest of the world. We ignore import tariffs on the grounds that they are a tiny fraction (less than 1%) of the value of goods imported into Virginia.

Eq.15. 
$$d_{t,i} = \bar{d}_{t,i} \left[ p_{t,i}^d \div \bar{p}_{t,i}^w \right]^{\eta_i^d} \quad \forall i \in I, t \in T$$

Description: See comments above.

Data: As with export demand we have used our judgement, combined with BEA data, to arrive at sensible estimates.

### **Import Demand**

Comments: Imports consist of the share of domestic consumption that is not supplied by domestic production.

Eq.16. 
$$m_{t,i} = (1 - d_{t,i}) x_{t,i} \quad \forall i \in I, t \in T$$

Description: See comments above.

## Average Prices by Industry

Comments: These aggregated prices are computed for each industry, and are weighted averages of the domestic price and the import price, with the weights consisting of the respective shares in consumption. The price is set to unity in the baseline situation.

$$\text{Eq.17.} \quad p_{t,i} = d_{t,i} p_{t,i}^d + (1 - d_{t,i}) \bar{p}_{t,i}^w \quad \forall i \in I, t \in T$$

## INVESTMENT

We first constructed a measure of the capital stock for each industrial sector for 2000. This stock, less depreciation and plus gross investment gives the capital stock for 2001. Gross investment is determined, sector-by-sector, based on the net of tax rate of return (relative to the return in the base period). For instance, once investment by the agricultural sector has been determined, it is transformed with the help of the capital coefficient matrix into the demand for goods and services for each sector in the economy.<sup>5</sup>

## Capital Stock

Comments: The capital stock in time  $t$  is the capital stock from the previous period adjusted for depreciation, and augmented by gross investment.

$$\text{Eq.18.} \quad u_{t,K,i} = u_{t-1,K,i} (1 - \delta_i) + n_{t,i} \quad \forall i \in I, t \in T$$

Description: See comments above.

Data: A complete discussion of the construction of capital stock figures is given in *Texas State Tax Modeling Program: Texas-STAMP* (1999); the same approach and the same data sources are used for Virginia.

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<sup>5</sup> The Capital Coefficient Matrix is a matrix of investments by using industries. It contains distribution ratios of new structures and equipment to using industries from the 1992 BEA capital flow tables.

## Gross Investment by Sector of Destination

Comments: The amount of gross investment in any given sector depends on the after-tax rate of return in that sector relative to the return in the base period. The terminology here can be confusing; investment destined for agriculture, for instance, consists of the purchases of goods that will add to the capital stock in the agricultural sector; the goods themselves will mainly come from other sectors (the sectors of source).

$$\text{Eq.19.} \quad n_{t,i} = \bar{n}_{t,i} \left[ \frac{r_{t,K,i} \left( 1 - \sum_{g \in GK} \tau_{t,g,K,i}^x \right) u_{t,K,i}}{\bar{r}_{t,K,i} \left( 1 - \sum_{g \in GK} \tau_{t,g,K,i} \right) \bar{u}_{t,K,i}} \right]^{\eta^i} \quad \forall i \in I, t \in T$$

Description: Gross investment is the baseline gross investment by industry adjusted to the change in after-tax capital rental rates.

Data: The rate of return is computed as the property-type income for each sector (from BEA) divided by the capital stock (authors' computations). Based on the econometric results from STAMP models estimated for Texas and elsewhere, we estimated the investment demand elasticity to be about 0.6.

## Gross Investment by Sector of Source

Comments: Given that investment has been determined for each sector of destination, this equation allows one to determine who will actually produce the investment goods. This is done with the help of a capital coefficient matrix.

$$\text{Eq.20.} \quad p_{t,i} \left( 1 + \sum_{g \in GS} \tau_{t,g,i}^n \right) c n_{t,i} = \sum_{i' \in I} \beta_{i,i'} n_{t,i'} \quad \forall i \in I, t \in T$$

Description: The gross investment by source in after-tax prices is a function of investment by destination according to the capital coefficient matrix.



Comments: Spending on these items is assumed to take a fixed fraction of total government receipts (from taxes and net intergovernmental transfers, less government savings). The endogenous sectors are state spending on education, health, safety, transport and “other,” and local spending on education and health.

Eq.23.

$$p_{t,i} \left( 1 + \sum_{g \in GS} \tau_{t,g,i}^g \right) cg_{t,i,g} = \alpha_{i,g} \left( y_{t,g} + \sum_{g' \in G} b_{t,g,g'} - \sum_{g' \in G} b_{t,g',g} + b_{t,usstx,g} - \sum_{h \in H} w_{t,h,g} a_{t,h}^n \tau_{t,h,g}^{pc} - \bar{s}_{t,g} \right)$$

$\forall i \in I, g \in GN, t \in T$

Description: The government spending in after-tax prices computed according to their share of government income plus net inter-government transfers less government savings and transfer payments. Note that only state and local governments are endogenous in the model.

Data: The shares of spending going to these sectors are based on an analysis of the spending patterns of state and local government in Virginia in 2002, the latest year for which sufficiently detailed data were available.

### **Government Endogenous Rental of Factors**

Comments: As in the case of goods and services, government is also assumed to devote a fixed share of its total spending to the purchase of labor and capital services for those sectors considered to be endogenous.

Eq.24.

$$u_{t,f,g}^d r_{t,f}^a r_{t,f,g} = \alpha_{f,g} \left( y_{t,g} + \sum_{g' \in G} b_{t,g,g'} - \sum_{g' \in G} b_{t,g',g} + b_{t,usstx,g} - \sum_{h \in H} w_{t,h,g} \alpha_{t,h}^n \tau_{t,h,g}^{pc} - \bar{s}_{t,g} \right) \quad \forall f \in F, g \in GN, t \in T$$

Description: The government factor demand is computed according to the share of each government in total government spending, including net inter-government transfers, less savings and transfer payments.

### Government Infrastructure Capital Stock

Comments: The government adds to its infrastructure capital stock through its spending on the government transportation sector, STTRAN.

$$\text{Eq. 25} \quad gk_{t+1} = gk_t (1 - \delta) + tn_{t+1,i} \quad t \in T$$

Description: The infrastructure capital stock for the current year is the infrastructure for the previous year, less depreciation plus the net spending on transportation by state and local governments.

Data: The initial measure of Virginia infrastructure capital stock for 2003 was derived by applying actual Virginia transportation spending during the period 1983 to 2003 to Cohen and Morrison, 2004 estimate. The depreciation rate is 0.01667 as derived from Munnell, 1990 estimate of 60 year average service life.

### Government Savings

Comments: Government saving is a residual, consisting of revenue less spending.

Eq.26. 
$$s_{t,g} = y_{t,g} - \sum_{i \in I} c g_{t,i,g} p_{t,i} \left( 1 + \sum_{g \in GS} \tau_{t,g,i}^g \right) - \sum_{f \in F} u_{t,f,g}^d r_{t,f,g} r_{t,f}^a \left( 1 + \sum_{g' \in GF} \tau_{t,f,g',g}^x \right) - \left( \sum_{h \in H} w_{t,h,g} a_{t,h}^n \tau_{hg}^{pc} \right) - \sum_{g' \in G} b_{t,g',g} + b_{t,usstx,g} + \sum_{g' \in G} b_{t,g,g'} \quad \forall g \in G, t \in T$$

Description: Government savings is the residual from government income, after spending and factor rental, transfer payments, plus net inter-governmental transfers.

### Distribution of Taxes to Spending and Transfers

Comments: Tax units, in this case those sectors collecting revenues, distribute some of their receipts to spending units, and others directly in the form of transfers to households. The matrix IGTD (in the miscellaneous input file) identifies which units pass on their revenues to other spending units, and the flows are recorded in this equation.

Eq.27. 
$$b_{t,g',g} = \mu_{t,g',g} \left( y_{t,g} - \left( \sum_{h \in H} w_{t,h,g} a_{t,h}^n \tau_{t,h,g}^{pc} - \bar{s}_{t,g} \right) \right) \quad \forall g, g' \in G$$

Description: The intra-fund accounting to distribute the government income, less transfer payments and savings.

Data: This equation is based on institutional arrangements in place in Virginia.

### Endogenous Distribution of Virginia Funds

Comments: This equation details the flows from state funds to state spending sectors and from state spending sectors to local spending sectors.

Eq.28. 
$$b_{t,g,g'} = \mu_{t,g,g'} \left( \sum_{g''} b_{t,g',g''} \right) \quad \forall g, g' \in G$$

Description: Some funds are fixed to the original share.

Data: Based on an analysis of the current pattern of state spending in Virginia.

### **State Personal Income**

Comments: This equation defines state personal income as earnings (from labor and capital) plus transfer payments.

$$\text{Eq.29.} \quad spi = \sum_{h \in H} y_{t,h} + \sum_{h \in H} \sum_{g \in G} w_{t,h,g} \alpha_{t,h}^n \tau_{hg}^{pc}$$

Description: State personal income is the sum of household income and government transfer payments.

### **MODEL CLOSURE**

#### **Labor Market Clearing**

Comments: Labor supply equals labor demand. For this to occur, the wage rate must adjust to bring about this market clearing.

$$\text{Eq.30.} \quad \sum_{h \in H} a_{t,h}^w = \left( \sum_{i \in I} u_{t,L,i}^d + \sum_{g \in G} u_{t,L,g}^d \right) \varepsilon$$

Description: Total working households equals the sum of private employment and government employment.

#### **Capital Market Clearing**

Comments: Capital markets also clear for each sector. In other words, demand for capital by industries equals supply of capital.

$$\text{Eq.31.} \quad u_{t,K,i}^s = u_{t,K,i}^d \quad \forall i \in I, t \in T$$

Description: See comments above.

### Goods Market Clearing

Comments: Domestic demand (intermediate, consumer, government and investment demand) plus exports less imports must equal domestic supply.

$$\text{Eq.32.} \quad q_{t,i} = x_{t,i} + e_{t,i} - m_{t,i} \quad \forall i \in I, t \in T$$

Description: See comments above.

### Domestic Demand Defined

Comments: These equations define domestic demand for each sector.

$$\text{Eq.33.} \quad x_{t,i} = v_{t,i} + \sum_{h \in H} c_{t,i,h} + \sum_{g \in G} c g_{t,i,g} + c n_{t,i} \quad \forall i \in I, t \in T$$

Description: Domestic demand is the sum of intermediate demand, household consumption, government consumption and investments.

### PIT for Non Income Tax Units

Comments: This equation sets the personal income tax for non-income tax units to zero; this is a technicality that ensures the solution to the model does not create income tax revenue in an inappropriate place.

$$\text{Eq.34.} \quad t_{t,g,h} = 0 \quad \forall h \in H, g \notin GI, t \in T$$

### Set Intergovernmental Transfers to Zero if Not in Original SAM

Comments: This is another housekeeping equation that ensures the solution to the model does not create inter-governmental transfers where they should not occur.

Eq.35.  $b_{t,g,g'} = 0 \quad \forall g, g' \in G, t \in T \quad \text{where } \bar{b}_{gg'} = 0$

### **Federal Social Security Transfers to Virginia**

Comments: Transfers paid to Virginia households from the Federal social security system are assumed to be mainly determined by the number of households in the state.

Eq.36. 
$$b_{t,h,USSTX} = \bar{b}_{t,h,USSTX} \times \left( \frac{\bar{a}_{t,h}^n}{a_{t,h}} \right)$$

Description: Transfer payments are adjusted by the change in nonworking households.

### **Fix Exogenous Federal Transfers to Households**

Federal transfers to households are assumed to vary with the number of households in the state.

Eq.37. 
$$b_{t,h,USNOND} = \bar{b}_{t,h,USNOND} \times \left( \frac{a_{t,h}^n}{\bar{a}_{t,h}^n} \right)$$

Description: Transfer payments are adjusted by the change in nonworking households.

### **Fix Goods and Services Demand by Exogenous Government Units**

Comments: The purchases of goods and services by some government sectors are considered to be exogenous to the model. This equation fixes these values.

Eq.38.  $cg_{t,i,g} = \bar{c}g_{t,i,g} \quad \forall i \in I, g \in GX, t \in T$

### **Fix Factor Rentals Paid by Exogenous Government Units**

Comments: The purchases of the services of labor and capital are considered to be exogenous to the model. This equation fixes these values.

Eq.39.  $u_{t,f,g}^d = \bar{u}_{t,f,g}^d \quad \forall f \in F, g \in GX, t \in T$

### **Fix Intersectoral Wage Differentials**

Comments: Although wage rates differ from sector to sector, these differentials are assumed to remain fixed, as set by this equation. Household labor supply responds to overall wage rates, and not to the wage rates in any particular sector.

Eq.40.  $r_{t,L,i} = \bar{r}_{t,L,i} \quad \forall i \in I, t \in T$

### Fix Government Rental Rate for Capital to Initial Level

Comments: For Virginia-STAMP, we have set these rental rates to zero, in the absence of viable information about the rental rates paid by government on the capital that it uses. However, the relevant equations are included, and so government rental rates could be incorporated in a future version of the model.

$$\text{Eq.41.} \quad r_{t,K,g} = \bar{r}_{t,K,g} \quad \forall g \in G, t \in T$$

### Fix Economy Wide Scalar for Capital

Comments: The model allows both for an overall cost of capital, and sector-specific returns. This equation sets the overall scalar to its original level, so that only the sector-specific returns vary endogenously.

$$\text{Eq.42.} \quad r_{t,K}^a = \bar{r}_{t,K}^a \quad \forall f \in F, t \in T$$

### Set Transfer Payments to Zero if Originally So

Comments: This equation ensures that if transfer payments to households were zero in the original social accounting matrix, they remain at zero.

$$\text{Eq.43.} \quad w_{t,h,g} = 0 \quad \forall h \in H, g \in GWX, t \in T \quad \text{where} \quad \bar{w}_{t,h,g} = 0$$

### OBJECTIVE FUNCTION

Comments: This equation measures utility over the entire period of the dynamic model as measured by the sum of state personal income discounted. The variable is of interest in its own right. However it also provides a convenient variable for GAMS to maximize (or minimize), because it is an unrestricted variable without a subscript.

$$\text{Eq. 44.} \quad U = \sum_{t \in T} \beta_t \text{state}y_t \quad t \in T$$

Description: Utility is defined as the net present value of future state personal income levels.

**APPENDIX: DEFINITIONS AND GLOSSARY OF TERMS**

**Summary of Set Names**

<b>Sets</b>	<b>Dimension</b>	<b>Math</b>	<b>GAMS</b>
Factors	2	$f \in F$	F
Governments - All	39	$g \in G$	G
Governments - Factor Taxes	6	$g \in GF$	GF
Governments - Per Household Taxes	8	$g \in GH$	GH
Governments - Income Taxes	2	$g \in GI$	GI
Governments - Capital Income Taxes	6	$g \in GK$	GK
Governments - Endogenous Spending	16	$g \in GN$	GN
Governments - Sales or Excise Taxes	11	$g \in GS$	GS
Governments - Endogenous Transfer Payments	1	$g \in GWN$	GWN
Governments - Exogenous Transfer Payments	4	$g \in GWX$	GWX
Governments - Exogenous Spending	6	$g \in GX$	GX
Households	7	$h \in H$	H
Industries	27	$i \in I$ or $j \in I$	I
All Social Accounting Matrix Accounts	77	$z \in Z$	Z

### Summary of Parameter Names

Parameters	Dimension	Math	GAMS
Input Output Coefficients	77 x 77	-	A(Z,Z1)
Domestic Input Output Coefficients	27 x 27	$\alpha_{ii}$	AD(Z,Z1)
Government Spending Shares of Net Income	39 x 39	$\alpha_{ie}, \alpha_{fe}$	AG(Z,G)
Factor Share Exponents in Production Function	2 x 27	$\alpha_{fi}$	ALPHA(F,I)
Initial Shares of Consumption	27 x 7	$\alpha_{ih}$	ALPHA(I,H)
Deductibility of Taxes	3 x 3	$\alpha_{eg}^t$	ATAX(G,G1)
Income Elasticities of Demand	27 x 7	$\beta_{ih}$	BETA(I,H)
Capital Coefficient Matrix	27 x 27	$\beta_{ii}$	CCM(I,J)
Depreciation Rate	27	$\delta_i$	DEPR(I)
Export Price Elasticities	27	$\eta_i^e$	ETA(E)
Domestic Demand Elasticity	27	$\eta_i^d$	ETAD(I)
Investment Supply Elasticity	1	$\eta_i$	ETAI
L Supply Elasticity with respect to Average Wage	7	$\eta_h^{ls}$	ETARA(H)
Labor Supply Elasticity with respect to TP's <sup>6</sup>	7	$\eta_h^{tp}$	ETATP(H)
Labor Supply Elasticity with respect to Taxes	7	$\eta_h^{PIT}$	ETAPIT(H)
Responsiveness of In-Migration to Unemployment	7	$\eta_h^u$	ETAU(H)
Responsiveness of In-Migration to Disp. Income	7	$\eta_h^{yd}$	ETAYD(H)
Production Function Scale	27	$\gamma_i$	GAMMA(I)
Types of Inter-Government Transfers	39 x 39	-	IGTD(G,G1)
Correction Factor between Households and Jobs	1	$\varepsilon$	JOBCOR
Cross-Price Elasticities	27 x 27	$\lambda_{ii}$	LAMBDA(I,J)
Miscellaneous Industry Parameters	27 x 10	-	MISC(Z,*)
Income Tax Table Data in Input File	7 x 8	-	MISCG(G,H,*)
Miscellaneous Household Parameters	7 x 8	-	MISCH(H,*)
Natural Rate of Population Growth	7	$\pi_h$	NRPG(H)
Substitution Exponent in Production Function	27	$\rho_i$	RHO(I)
Social Accounting Matrix	77 x 77	$\square_{zz}$	SAM(Z,Z1)
Consumption Sales and Excise Tax Rates	9 x 27	$\tau_{oi}^c$	TAUC(G,I)
Factor Tax Rates	5 x 2 x 77	$\tau_{ofz}$	TAUF(G,F,Z)
Factor Taxes applied to Factors	5 x 2	-	TAUFF(GF,G)
Employee Portion of Factor Taxes	5 x 2	$\tau_{of}$	TAUFH(G,F)
Experimental Factor Tax Rates	5 x 2 x 77	$\tau_{ofz}^x$	TAUFX(G,F,Z)
Government Sales and Excise Tax Rates	9 x 27	$\tau_{ei}^g$	TAUG(G,I)
Household Taxes other than PIT	1 x 7	$\tau_{oh}$	TAUH(G,H)
Investment Sales and Excise Tax Rates	9 x 27	$\tau_{oi}^n$	TAUN(G,I)
Sales and Excise Tax Rates	9 x 27	$\tau_{oi}^q$	TAUQ(G,I)
Intermediate Good Sales and Excise Tax Rates	9 x 27	$\tau_{oi}^v$	TAUV(G,I)
Tax Bracket Base Amount	2 x 7	$\tau_{oh}^b$	TAXBASE(G,H)
Tax Bracket Minimum Taxable Earnings	2 x 7	$\tau_{gh}^d$	TAXB(M,G,H)
Tax Constant to Correct Calculated to Observed	2 x 7	$\tau_{oh}^c$	TAXCVC(G,H)
Tax Deduction other than Standard and other PIT	2 x 7	$\tau_{oh}^o$	TAXOD(G,H)
Percentage Itemizing	2 x 7	$\tau_{gh}^i$	TAXPI(G,H)
Tax Destination Shares	39 x 39	$\mu_{oo'}$	TAXS(G,G1)
Tax Deduction for Standard Deductions	2 x 7	$\tau_{oh}^s$	TAXSD(G,H)
Percent of Households Receiving TP's	7 x 6	$\tau_{hp}^{pc}$	TPC(H,G)

<sup>6</sup> TP is abbreviation for transfer payments.

### Summary of Variable Names

Variables	Dimension	Math	GAMS
Public Consumption	27 x 39	$c_{ig}$	CG(I,G)
Private Consumption	27 x 7	$c_{ih}$	CH(I,H)
Gross Investment by Sector of Source	27	$c_{in}$	CN(I)
Consumer Price Index	7	$p_h$	CPI(H)
Exports	27	$e_i$	CX(I)
Domestic Share of Domestic Consumption	27	$d_i$	D(I)
Domestic Demand	27	$x_i$	DD(I)
Domestic Supply	27	$q_i$	DS(I)
Sectoral Factor Demand	2 x 77	$u_{fi}^d, u_{fg}^d$	FD(F,Z)
Number of Households	7	$a_h$	HH(H)
Number of Non-Working Households	7	$a_h^n$	HN(H)
Number of Working Households	7	$a_h^w$	HW(H)
Household Out-Migration	7	$a_h^o$	MO(H)
Household In-Migration	7	$a_h^i$	MI(H)
Inter-Governmental Transfers	37 x 37	$B_{gg'}$	IGT(G,G1)
Capital Stock	27	$u_{Ki}^s$	KS(I)
Imports	27	$m_i$	M(I)
Gross Investment by Sector of Destination	27	$n_i$	N(I)
Net Capital Inflow	1	$z$	NKI
Aggregate Price	27	$p_i$	P(I)
Aggregate Price including Sales/Excise Taxes	27	$p_i^c$	PC(I)
Domestic Producer Price	27	$p_i^d$	PD(I)
Per Household Personal Income Taxes	2 x 7	$t_{gh}$	PIT(G,H)
Producer Price Index	1	$p$	PPI
Value Added Price	27	$p_i^{va}$	PVA(I)
World Price (Rest of US and Rest of World)	27	$p_i^w$	PW(I)
Sectoral Factor Rental Rates	2 x 27	$r_{fi}, r_{fg}$	R(F,I)
Economy Wide Scalar for Factor Rental Rates	2	$r_f^a$	RA(F)
Government Savings	39	$s_g$	S(G)
Private Savings	7	$s_h$	S(H)
State Personal Income	1	$q$	SPI
Transfer Payments	7 x 39	$w_{hg}$	TP(H,G)
Intermediate Goods	27	$v_i$	V(I)
Factor Income	2	$y_f$	Y(F)
Government Income	39	$y_g$	Y(G)
Household Income	7	$y_h$	Y(H)
Household after Tax Income including TP's	7	$Y_h^d$	YD(H)